Software Performance Engineering
SWEN 549

Example Queuing Theory Problems

Name: ____

These questions are similar to what will be on the exam. Use these to study for the exam, but you do not need to hand them in.

1. (10 points) Suppose we have a bartender handling customers who are arriving at 7 customers per hour, following a Markovian process. What is the probability that a customer will be served within a half hour?

Solution:

$\lambda = 7$

$x = 0.5$

$F_7(0.5) = 1 - e^{-7/2} = 96.98\%$

2. (10 points) Suppose we have a librarian handling patrons who are arriving every 20 minutes, following a Markovian process. What is the probability that a patron will have to wait more than an hour?

Solution:

by minutes...

$1/\lambda = 20$, so $\lambda = 1/20 = 0.05$

$1 - F_{1/20}(60) = e^{-60/20} = 4.9\%$

or, by hour...

$1/\lambda = 1/3$, so $\lambda = 3$ customers per hour

$1 - F_3(1) = e^{-3} = 4.9\%$

3. (10 points) Suppose we want to ensure that 85% of customers at an insurance agency should not have to wait longer than 15 minutes. How many customers per hour must the agency be able to process, on average, to achieve this goal?

Solution:

15 minutes is 1/4 an hour...

$F_\lambda(1/4) \leq 0.85$

$1 - e^{-\lambda/4} \leq 0.85$

$1 - 0.85 \leq e^{-\lambda/4}$

$0.15 \leq e^{-\lambda/4}$

$ln(0.15) \leq ln(e^{-\lambda/4})$

$ln(0.15) \leq -\lambda/4$

$-4ln(0.15) > \lambda$

$\lambda > 7.5$ customers per hour

4. (10 points) The checkout at a bank has a customer arrival rate of 30 customers per hour, and they have a service rate of 40 customers per hour. What is the mean queue length? Assume both of these random variables are Markovian, and that the entire bank is handled by one queue.

Solution:

$\rho = 30/40 = 0.75$

$\frac{0.75}{0.25} = 3$

5. (10 points) The checkout at a game store has an average of 5 minutes between each customer arrival. You want to have a mean queue length of at most 2 customers. What mean service rate per hour will we need to achieve this?

(Assume both arrival rate and service rate variables are Markovian.)

Solution:

convert to hours initially

$\frac{1}{\lambda} = \frac{5}{60}$, so $\lambda = 60/5 = 12$

$\frac{\rho}{1-\rho} \leq 2$

$\rho \leq 2 - 2\rho$

$3\rho \leq 2$

$\rho \leq 2/3$

$\rho = \frac{\lambda}{\mu}$

$\frac{2}{3} = \frac{12}{\mu}$

$\lambda > 18$ customers per hour

Or... keep it all in minutes until the end...

$\frac{1}{\mu} = 5$, so $\mu = 1/5$ customers / hour

(same logic above)

$\rho \leq 2/3$

$\rho = \frac{\lambda}{\mu}$

$\frac{2}{3} = \frac{0.2}{\mu}$

$2\mu = 0.6$

$\mu = 0.3$ customers / minute

Or $0.3 * 60 = 18$ customers / hour

6. (10 points) The checkout at a cigar shop has a customer arrival rate of 4 customers per hour. You currently have a service rate of 6 customers per hour, but would like to make your mean

response time 70% of what it is now. By what percentage will you need to improve your current service rate to achieve this goal?

(Assume both arrival rate and service rate variables are Markovian.)

Solution:

First find the current response time...

$\rho = 4/6 = 0.\bar{6}$

$\bar{n} = \frac{\rho}{1-\rho} = 2$

In this system, $X = \lambda = 4$ because we are able to process everybody. So then we apply Little's Law:

$4R = 2$ or $R = 0.5$ hours

But! We want to improve our R, so...

$0.70R = 0.35$ hours

$\bar{n} = 0.35 * 4 = 1.4$ is the desired queue length

$1.4 = \frac{\rho}{1-\rho}$

$1.4 - 1.4\rho = \rho$

$1.4 = 2.4\rho$

$\rho = \frac{1.4}{2.4} = 0.58\bar{3}$ is our desired ρ

$0.58\bar{3} = \frac{\lambda}{\mu} = \frac{4}{\mu_{improved}}$

$\mu_{improved} = \frac{4}{0.58\bar{3}} = 6.857$ customers per hour

Percentage improvement d is $\frac{\mu_{improved}}{\mu_{old}} = \frac{6.857}{6} = 1.1428$

$d = 1.1428$ or 14.3%

7. (10 points) Suppose you have a central server system with a CPU, and IO devices of an SSD and HDD. The CPU, SSD, and HDD all have mean service rates of 2 jobs/ms, 1 job/ms, and 0.6 job/ms, respectively. The global throughput of the system is 0.65 jobs/ms. The utilization of the CPU is measured at 68.25% and the utilization of the SSD is measured at 39%. What is the expected utilization of the HDD?

Solution:

We were given μ for each, so convert to S.

$S_{cpu} = 1/2 = 0.5$, $S_{ssd} = 1$, $S_{hdd} = 10/6 = 1.\bar{6}$

Now use $U_i = X_{global} D_i$

$0.6825 = 0.65 D_{cpu}$, so $D_{cpu} = 0.6825/0.65 = 1.05$

$0.39 = 0.65 D_{ssd}$, so $D_{ssd} = 0.35/0.6 = 0.6$

Now use $D_i = V_i S_i$

$0.5 * V_{cpu} = 1.05$, so $V_{cpu} = 2.1$

$1 * V_{ssd} =$, so $V_{ssd} = 0.6$

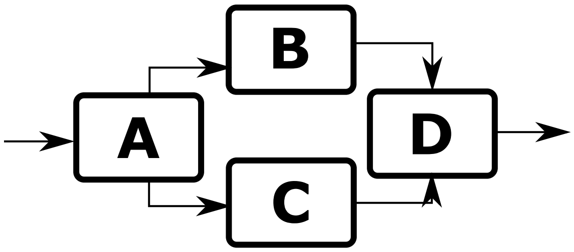
And we know that $V_{cpu} = 1 + V_{ssd} + V_{hdd}$

$2.1 = 1 + 0.6 + V_{hdd}$, so $V_{hdd} = 0.5$

Use $U_i = X_{global} D_i$, i.e. $U_{hdd} = X_{global} V_{hdd} S_{hdd}$

$U_{hdd} = 0.65 * 0.5 * (10/6) = 54.1\bar{6}\%$

8. (10 points) Suppose we have a system similar to the central server system, but with the following structure:



Now suppose that the mean service times for A,B,C, and D are: 0.5, 0.6, 0.7, and 0.4 ms/job respectively. The B queue is visited three times as often as the C queue. The C queue is visited half as many times as the A queue. What is the maximum throughput of this system?

Solution:

Based on the diagram, we can assume that the visit ratios $V_A = V_D$, and that $V_A = 2 + V_B + V_C$

And since we know from the problem that:

$V_B = 3V_C$ and

$V_C = 0.5V_A$

Then plugging in we get $V_A = 2 + 4V_C = 4$

So $V_D = 4$

$V_C = 2$

$V_B = 6$

Multiply those by the service times to get demands:

$D_A = 4 * 0.5 = 2$

$D_B = 6 * 0.6 = 3.6$

$D_C = 2 * 0.7 = 1.4$

$D_D = 4 * 0.4 = 1.6$

Thus, $D_{max} = 3.6$

So $X_{global} < \frac{1}{3.6} = 0.2\bar{7}$ jobs/ms